

$\rho\omega$ -Mixing and the Pion Electromagnetic Form Factor in the Nambu-Jona-Lasinio Model

R. Friedrich^{1,2} and H. Reinhardt¹

*Institut für Theoretische Physik, Universität Tübingen,
Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

Abstract

The $\rho\omega$ -mixing generated by the isospin breaking of the current quark masses $m_u \neq m_d$ is studied within the bosonized NJL model in the gradient expansion. The resulting effective meson lagrangian naturally incorporates vector meson dominance. By including pion loops an excellent description of both the pion electromagnetic form factor and of the $\pi^+\pi^-$ phase shifts in the vector-isovector channel is obtained. The $\rho\omega$ -mixing can be treated in the static approximation but is absolutely necessary to reproduce the fine structure of the electromagnetic form factor, while the pion loops are necessary to obtain the correct energy dependence of the phase shifts.

¹Supported in part by *COSY* under contract 41170833

² e-mail: friedric@ptdec5.tphys.physik.uni-tuebingen.de

1 Introduction

Since the success of current algebra it has been known that the low energy meson dynamics is dominated by chiral symmetry and its spontaneous and explicit breaking. This is the conceptual basis for chiral perturbation theory [1] where the a priori unknown effective meson theory underlying QCD is systematically expanded in powers of derivatives of the meson fields and the arising expansion coefficients are considered as phenomenological parameters to be determined from the data.

The mechanism of spontaneous breaking of chiral symmetry with the appearance of the pseudoscalar mesons as Goldstone bosons has been well understood in a microscopic fashion within the bosonized Nambu–Jona-Lasinio model [2]. The gradient expansion of the bosonized NJL model essentially reproduces the chiral perturbation expansion and expresses the various expansion coefficients of chiral perturbation theory in terms of a few microscopic parameters of the NJL model. Furthermore the bosonized NJL model also shows how the explicit symmetry breaking by the small current quark masses gives rise to finite masses of the pseudoscalar mesons. The current quark masses do explicitly break not only chiral symmetry but in addition also charge or isospin symmetry. This breaking of charge symmetry induces various meson mixings which have recently received increasing interest. For a comprehensive review see for example [3].

Given the success of the bosonized NJL model in the isospin symmetric case one can expect that this model also accounts for the isospin symmetry breaking effects like the $\rho\omega$ -mixing, since the explicit flavor symmetry breaking through the current quark masses is included in this model. Some attempts in this direction have been undertaken in [4] where the NJL model with a sharp ultraviolet momentum cutoff was studied in lowest order gradient expansion. However a sharp cutoff spoils gauge invariance, which becomes important when electromagnetic processes are considered, and as a consequence the Ward identities are violated and electric charge is not conserved.

So far the bosonized NJL model has been mainly used in the gradient expansion on tree level, which has proved already sufficient to reproduce the gross feature of the low energy meson data. In this paper we want to push the model even further and study processes where on the one hand one has to go beyond the tree approximation and include pion loops and on the other hand has to include the subtle effects of meson mixings induced by the small charge symmetry breaking of the current quark mass. For this purpose we shall study the electromagnetic form factor of the pion and the $\pi^+\pi^-$ phase shifts in the vector–isovector channel. We will show that both the form factor and the phase shifts can be quite satisfactorily reproduced within the bosonized NJL model in leading order gradient expansion provided one includes the $\rho\omega$ -mixing and pion loops. For a quantitative analysis it is important to describe both processes simultaneously. According

to vector meson dominance³ they are both determined by the ρ -propagator, which is dressed by pion loops and modified by the $\rho\omega$ -mixing. While the pion loops are necessary to reproduce the correct energy dependence of the phase shifts the $\rho\omega$ -mixing will show up in the fine structure of the electromagnetic form factor.

The organization of the paper is as follows: In the next section we define the NJL model and present the effective meson lagrangian resulting in leading order gradient expansion after bosonization. In particular we derive the $\rho\omega$ -mixing induced by the charge symmetry breaking of the current quark masses. In section 3 we calculate the effective ρ meson propagator which contains besides the $\rho\omega$ -mixing also pion loops. Section 4 is devoted to the numerical results.

2 The NJL Model and the effective meson lagrangian

The NJL model [5] was originally introduced as an effective theory for the nucleon field. Subsequently, it had a revival as a model for the low-energy quark flavor dynamics. The two-flavor NJL model is defined by the following lagrange density

$$\begin{aligned} \mathcal{L} = & \bar{q}(i\cancel{D} - \hat{m}_0)q + \frac{G_1}{2} \left((\bar{q}\tau^a q)^2 + (\bar{q}i\gamma_5\tau^a q)^2 \right) \\ & - \frac{G_2}{2} \left((\bar{q}\gamma^\mu\tau^a q)^2 + (\bar{q}\gamma^\mu\gamma_5\tau^a q)^2 \right) . \end{aligned} \quad (1)$$

Here $q = (u, d)^T$ denotes the quark field, τ^a are the isospin Pauli matrices, $\hat{m}_0 = \text{diag}(m_0^u, m_0^d)$ is the current quark matrix and G_1 and G_2 are coupling constants of dimension $(\text{MeV})^{-2}$. For $\hat{m}_0 = 0$ the model has $\text{SU}(2)_R \times \text{SU}(2)_L$ chiral symmetry. Note that chiral symmetry allows for independent coupling constants in the scalar-pseudoscalar and vector-axialvector sectors.⁴

The generating functional for Green's functions of quark bilinears is given by the path integral

$$Z(\eta) = \int \mathcal{D}q \mathcal{D}\bar{q} e^{i \int d^4x (\mathcal{L}(q) + \mathcal{L}_{\text{source}}(j))} . \quad (2)$$

Here we have included a source term

$$\mathcal{L}_{\text{source}}(j) = j(\bar{q}\Gamma q) + h.c. , \quad (3)$$

where Γ is a suitable combination of Dirac and isospin matrices.

Applying by now standard bosonization techniques [2] one converts the NJL model (1) into an effective meson theory, which can be cast into the following form [6]

³The bosonized NJL model naturally accounts for vector meson dominance [2].

⁴If the interaction in (1) is deduced from one gluon exchange by Fierz transformation one obtains $G_2 = \frac{1}{2}G_1$.

$$\begin{aligned}
\int \mathcal{L} = & -i\text{Tr} \ln (i\partial - \Sigma + V + A\gamma_5) \\
& - \frac{1}{4G_1} \text{tr}_F \int [(\Sigma)^2 - \widehat{m}_0(\xi_L^\dagger \Sigma \xi_R + \xi_R^\dagger \Sigma \xi_L)] \\
& + \frac{1}{4G_2} \text{tr}_F \int [(V_\mu - \mathcal{V}_\mu^\pi)^2 + (A_\mu - \mathcal{A}_\mu^\pi)^2] + S_{anom}
\end{aligned} \tag{4}$$

Here $V_\mu = V_\mu^a \tau^a$ and $A_\mu = A_\mu^a \tau^a$ denote the vector and axialvector fields, respectively. Furthermore Σ is a scalar field (the so called chiral radius) and $\xi_{L,R}$ are unitary fields, which are related to the more standard scalar $s = s^a \tau^a$ and pseudoscalar $p = p^a \tau^a$ fields by

$$s + ip = \xi_L^\dagger \Sigma \xi_R \quad . \tag{5}$$

Moreover we have introduced the induced vector and axialvector fields

$$\begin{aligned}
\mathcal{V}_\mu^\pi &= \frac{i}{2} (\xi_R \partial_\mu \xi_R^\dagger + \xi_L \partial_\mu \xi_L^\dagger) \\
\mathcal{A}_\mu^\pi &= \frac{i}{2} (\xi_R \partial_\mu \xi_R^\dagger - \xi_L \partial_\mu \xi_L^\dagger) \quad ,
\end{aligned}$$

which originate from a chiral rotation of the original quark fields (see ref. [2]). The corresponding Jacobian [7] yields the (integrated) chiral anomaly S_{anom} , the explicit form of which we do not need in the following. (In leading order gradient expansion S_{anom} is given by the Wess–Zumino action [8].) Under a chiral transformation $\text{SU}(2)_R \times \text{SU}(2)_L$ of the quarks

$$q_L \rightarrow L q_L \quad q_R \rightarrow R q_R$$

the meson fields transform according to

$$\begin{aligned}
\xi_R(x) &\rightarrow h(x) \xi_R(x) R^\dagger \quad , \quad \xi_L(x) \rightarrow h(x) \xi_L(x) L^\dagger \quad , \\
\Sigma &\rightarrow h(x) \Sigma h^\dagger(x) \quad , \\
V^\mu &\rightarrow h(x) V^\mu h^\dagger(x) + i h(x) \partial^\mu h^\dagger(x) \quad , \\
A^\mu &\rightarrow h(x) A^\mu h^\dagger(x) \quad ,
\end{aligned}$$

where $h(x)$ is an element of the so called local hidden symmetry group [9]. Due to this symmetry the effective action (4) depends on $\xi_{L,R}$ only via the chiral field

$$U(x) = \xi_L^\dagger(x) \xi_R(x) \quad .$$

To remove this extra gauge symmetry we adopt the unitary gauge

$$\xi_R = \xi_L^\dagger = \xi \quad . \tag{6}$$

The quark loops is a diverging object and needs regularization. We will use the proper time regularization throughout the paper.

In the vacuum defined by the stationary points of the effective action, only the scalar field Σ develops a nonzero expectation value

$$\Sigma = \hat{m} = \text{diag}(m_u, m_d) \quad , \quad (7)$$

which represents the constituent quark mass m_i , $i = u, d$ and signals the spontaneous breakdown of chiral symmetry. Stationarity of the action with respect to variation in Σ yields the following gap equation

$$m_i = m_i^0 + G_1 \frac{N_c}{2\pi^2} (m^i)^3 \Gamma(-1, (m^i)^2/\Lambda^2) \quad , \quad (8)$$

whose solutions yield flavor dependent constituent quark masses m_i , $i = u, d$. Hence the vacuum configuration m_i , $i = u, d$ breaks already charge symmetry.

The physical mesons are given by the small amplitude excitations of the meson fields around their vacuum value. We shall concentrate on the low lying non-strange mesons, which are the π, ρ and ω , and put the scalar field Σ on its vacuum value. The contact terms in the effective meson action will give rise to the meson masses. The quark loop depends, with Σ fixed at its vacuum value, only on the vector and axialvector fields V_μ and A_μ . Expansion of the quark loop in powers of V_μ and A_μ yields in second order:

$$\begin{aligned} -i\text{Tr} \ln (i\cancel{\partial} - \Sigma + \cancel{V} + \cancel{A}\gamma_5) &= -i\text{Tr} \ln (i\cancel{\partial} - \Sigma) \\ &+ \frac{1}{2} \int V_a^\mu(x) \Pi_{\mu\nu}^{V\,ab}(x, y) V_b^\nu(y) \\ &+ \frac{1}{2} \int A_a^\mu(x) \Pi_{\mu\nu}^{A\,ab}(x, y) A_b^\nu(y) \end{aligned} \quad (9)$$

where the meson self energy is given by

$$\Pi_{\mu\nu}^{(V,A)\,ab}(x, y) = \frac{-i}{2} \text{Tr} \left(G_0(y, x) \gamma_\mu \begin{Bmatrix} 1 \\ \gamma_5 \end{Bmatrix} \tau^a G_0(x, y) \gamma_\nu \begin{Bmatrix} 1 \\ \gamma_5 \end{Bmatrix} \tau^b \right) \quad (10)$$

Following [2] we will perform a gradient expansion

$$\Pi_{\mu\nu}^{(V,A)\,ab}(p^2) = \Pi_{\mu\nu}^{(V,A)\,ab}(0) + \left(\frac{d}{dp^2} \Pi_{\mu\nu}^{(V,A)\,ab}(p^2) \right)_{p^2=0} + \dots \quad (11)$$

For the charge symmetric case extraction of the meson properties in the gradient expansion of the effective meson theory has been performed in [2]. After redefinition of the meson fields to eliminate the πa_1 -mixing and to bring the resulting effective meson lagrangian in the standard form one finds the following results [6]

$$\begin{aligned} M_\pi^2 &= \frac{m_0 m}{G_1 F_\pi^2} \quad , \quad M_V^2 = \frac{g_V^2}{4G_2} \quad , \quad M_A^2 = M_V^2 + 6m^2 \quad , \\ F_\pi^2 &= \frac{1}{4G_2} \left(1 - \frac{M_V^2}{M_A^2} \right) \quad , \quad g_{V\pi\pi} = \frac{g_V}{8G_2 F_\pi^2} \quad , \end{aligned} \quad (12)$$

with

$$g_V^{-2} = \frac{N_c}{24\pi^2} \Gamma \left(0, \frac{\Sigma_0^2}{\Lambda^2} \right) \quad \Sigma_0 = \frac{1}{2}(m_u + m_d) \quad . \quad (13)$$

Here we shall go beyond [2, 6] and include in addition the charge symmetry breaking which gives rise to the $\rho\omega$ -mixing. As well as the dressing of the ρ propagator by pion loops. The analogous $a_1 a_D$ -mixing is experimentally less understood. For this purpose we ignore the a_1 channel in the following (after the πa_1 -mixing has been properly removed). When the isospin symmetry breaking part $\Delta\Sigma$ defined by

$$\Sigma = \Sigma_0 \tau_0 + \Delta\Sigma \tau_3 \quad , \quad \Sigma_0 = \frac{1}{2}(m_u + m_d) \quad , \quad \Delta\Sigma = \frac{1}{2}(m_u - m_d) \quad (14)$$

is included the vector meson self energy Π^V contains besides the flavor diagonal parts considered already in [2] also an isospin mixing part $\Pi_{\mu\nu}^{V03}(p^2)$ which gives rise to a $\rho\omega$ -mixing. Note also that the vector meson mass terms do not lead to any $\rho\omega$ -mixing. So the total mixing $\mathcal{L}_{\rho\omega}$ comes entirely from the quark loop

$$\int \mathcal{L}_{\rho\omega} = \int \rho_\mu^3 \Pi_{\mu\nu}^{V03} \omega_\nu \quad (15)$$

From Lorentz invariance we expect the mixing term to be of the form (in momentum representation)

$$\mathcal{L}_{\rho\omega} = -m_{\rho\omega}^2(p^2) \rho_\mu^3 \omega^\mu + f(p^2) p^\mu \omega_\mu p^\nu \rho_\nu^3 \quad . \quad (16)$$

The last term contains the derivative couplings of ρ and ω . This term will not contribute to the electromagnetic form factor of the pions where the ρ couples to the conserved electromagnetic current. In fact by vector meson dominance we have $j_\mu^{em} \sim \rho_\mu$ and $\partial^\mu j_\mu^{em} = 0$ implies $\partial^\mu \rho_\mu = 0$.

From equation (10) the mixing term can be straight forwardly evaluated. Working out the isospin trace shows that Π^{V03} is given by the difference between the u and d vector quark loops

$$\Pi_{\mu\nu}^{V03}(x, y) = \frac{-i}{2} \text{Tr} (G_u(x, y) \gamma_\mu G_u(y, x) \gamma_\nu - (u \rightarrow d)) \quad (17)$$

Using here for the quark loop again the proper time regularization one finds

$$\begin{aligned} -m_{\rho\omega}^2 \rho_\mu^3 \omega^\mu + f p^\mu \rho_\mu^3 p^\nu \omega_\nu &= iN_c \int \frac{d^4 k}{(2\pi)^4} \text{tr} \frac{\not{k} - \not{p} + m_u}{(k-p)^2 - m_u^2} \not{p}^3 \frac{\not{k} + m_u}{k^2 - m_u^2} \not{p} - \\ &\quad (m_u \rightarrow m_d) \\ &= \frac{N_c}{2\pi^2} \left(p^2 \rho_\mu^3 \omega^\mu - p^\mu \rho_\mu^3 p^\nu \omega_\nu \right) \int_0^1 dx x(1-x) \times \\ &\quad \left\{ \Gamma \left(0, \frac{(m_u^2 - x(1-x)p^2)}{\Lambda^2} \right) - \Gamma \left(0, \frac{(m_d^2 - x(1-x)p^2)}{\Lambda^2} \right) \right\} \end{aligned} \quad (18)$$

from which we identify the $\rho\omega$ -mixing

$$m_{\rho\omega}^2(p^2) = 2p^2 \frac{\int_0^1 dx x(1-x) \Gamma \left(0, \frac{m_d^2 - x(1-x)p^2}{\Lambda^2} \right) - \Gamma \left(0, \frac{m_u^2 - x(1-x)p^2}{\Lambda^2} \right)}{\int_0^1 dx x(1-x) \Gamma \left(0, \frac{\Sigma_0^2 - x(1-x)p^2}{\Lambda^2} \right)} . \quad (19)$$

Let us emphasize that

$$m_{\rho\omega}^2(p^2 = 0) = 0 \quad , \quad (20)$$

which is a consequence of the gauge symmetry preserving regularization method used. If we had used another regularization scheme, e.g. a sharp Euclidean cutoff, we would have got also a term which does not vanish for $p^2 = 0$ [4]. The gradient expansion of the $\rho\omega$ -mixing yields in leading order

$$m_{\rho\omega}^2(p^2) = 2p^2 \frac{\Gamma \left(0, \frac{m_d^2}{\Lambda^2} \right) - \Gamma \left(0, \frac{m_u^2}{\Lambda^2} \right)}{\Gamma \left(0, \frac{\Sigma_0^2}{\Lambda^2} \right)} . \quad (21)$$

On the mass shell $p^2 = M_\rho^2 \approx M_\omega^2 \approx 0.6 \text{ GeV}^2$ this mixing strength is empirically known to be $m_{\rho\omega}^2 = (4520 \pm 600) \text{ MeV}^2$. Fixing all parameters of the NJL model except for m_d in the isospin symmetric sector (see [2]) the empirically value requires $m_d - m_u \approx (1-2) \text{ MeV}$ for $m_u = 300 \text{ MeV}$.

In the bosonized NJL model all quark observables become functionals of the meson fields. It is straight forward to derive the corresponding expressions for the quark currents by including

appropriate sources j according to (3) in the original quark theory (2) and taking at the end derivatives with respect to the sources in the bosonized theory. For the electromagnetic current one finds

$$j_\mu^{elm} = \frac{M_\rho^2}{g_V} \rho_\mu^3 + \frac{1}{3} \frac{M_\omega^2}{g_V} \omega_\mu + \frac{\sqrt{2}}{3} \frac{M_\Phi^2}{g_\Phi} \Phi_\mu \quad , \quad (22)$$

which is a manifestation of the vector meson dominance hypothesis according to which the photon couples to hadrons via the vector mesons. The vanishing of the $\rho\omega$ -mixing at $p^2 = 0$ (see equation (20)) implies that an onshell photon can couple to the pion only via the ρ -meson since only the ρ -meson couples in the effective meson lagrangian (4) to the pionic vector current \mathcal{V}^μ .

3 The effective ρ propagator

Using the leading order gradient expansion and ignoring the axial vector mesons (after its mixing with the π -field is removed) the effective meson lagrangian obtained in the previous section by expanding the quark loop up to the second order in the vector meson and pion fields is given by

$$\mathcal{L} = \mathcal{L}_0^\rho + \mathcal{L}_0^\pi + \mathcal{L}_0^\omega + \mathcal{L}_{\rho\omega} + \mathcal{L}_{\rho\pi} \quad , \quad (23)$$

Here

$$\mathcal{L}_0^\rho = -\frac{1}{4} \left(\partial_\mu \rho_\nu^a - \partial_\nu \rho_\mu^a \right)^2 + \frac{1}{2} m_\rho^2 \rho_\mu^{a2} \quad , \quad (24)$$

$$\mathcal{L}_0^\omega = -\frac{1}{4} \left(\partial_\mu \omega_\nu - \partial_\nu \omega_\mu \right)^2 + \frac{1}{2} m_\omega^2 \omega_\mu^2 \quad , \quad (25)$$

$$\mathcal{L}_0^\pi = \frac{1}{2} \left(\partial_\mu \pi^a \right)^2 - \frac{1}{2} m_\pi^2 \pi^{a2} \quad . \quad (26)$$

are the free lagrangians of the ρ , ω and π and

$$\mathcal{L}_{\rho\omega} = m_{\rho\omega}^2 \left(p^2 = m_\omega^2 \right) \rho_\mu^3 \omega^\mu \quad . \quad (27)$$

is the $\rho\omega$ -mixing, which has been taken on the mass shell $p^2 = m_\omega^2$ and

$$\mathcal{L}_{\rho\pi} = i g_{\rho\pi\pi} \epsilon^{abc} \rho_\mu^a \pi^b \partial^\mu \pi^c \quad (28)$$

arises from the expansion of the vector current in (4) in leading order in the pion field.⁵ We have ignored here the contribution from the chiral anomaly S_{anom} . The Wess-Zumino-action yields the leading term where ω_μ couples to the topological current B_μ

$$\int \omega_\mu B^\mu \quad , \quad B^\mu = \frac{1}{24\pi^2} \epsilon^{\mu\nu\kappa\lambda} \text{tr} L_\nu L_\kappa L_\lambda \sim \epsilon^{\mu\nu\kappa\lambda} \epsilon^{abc} \partial_\nu \pi^a \partial_\kappa \pi^b \partial_\lambda \pi^c \quad . \quad (29)$$

⁵One could as well keep the full momentum dependence of the $\rho\omega$ -mixing. This would however yield almost identical results due to the weakness of the $\rho\omega$ mixing, which matters only at the ρ -pole.

This term would contribute to the ω -propagator only through two-loop pion diagrams. In accord with the counting of chiral perturbation theory where vector meson loops are subleading to pion loops we will treat the effective meson lagrangian in tree approximation concerning the vector mesons but include the one pion loops. Thereby we will concentrate on the electromagnetic form factor of the pion and the $\pi^+\pi^-$ phase shifts in the vector-isovector channel, which both are exclusively determined by the ρ -propagator. The electromagnetic form factor of the pion $F_{em}(q^2)$ is directly related to the propagator of the ρ -meson $D_\rho(k^2)$ by

$$|F_{em}(k^2)| \propto |D_\rho(k^2)| , \quad (30)$$

which shows that the form factor directly probes the momentum dependence of the ρ -propagator. The missing normalization factor is determined by charge conservation

$$F_{em}(k^2 = 0) = 1 . \quad (31)$$

While the electromagnetic form factor obviously measures only the module of $D_\rho(k^2)$ the phase of the propagator (without $\rho\omega$ -mixing) is measured by the pion phase shifts in the vector-isovector channel. For the scattering amplitude a_1^1 one has

$$a_1^1 \propto \exp(i\delta_1^1) \sin(\delta_1^1) \propto D_\rho , \quad (32)$$

which yields for the phase shift δ_1^1

$$\tan(\delta_1^1) = -\frac{\text{Im}D_\rho^{-1}}{\text{Re}D_\rho^{-1}} . \quad (33)$$

Figure 1: The dressing of the ρ -propagator by pion loops. The curly lines denote the dressed ρ -propagator (thick curly line) and the free ρ -propagator (thin curly line), respectively.

In following we therefore calculate explicitly the ρ -propagator. We include the one pion loop to the ρ -propagator as shown in figure (1). It gives rise to a momentum dependent self energy of the ρ -meson and provides a finite width:

$$\begin{aligned} \Sigma_\rho^{\mu\nu}(p) &= \frac{ig_{\rho\pi\pi}^2}{2} \int \frac{d^4k}{(2\pi)^4} \frac{(2p-k)^\mu}{(k-p)^2 - m_\pi^2} \frac{(2p-k)^\nu}{k^2 - m_\pi^2} \\ &= \frac{g^2}{16\pi^2} \int_0^1 dx \left\{ 2\delta^{\mu\nu} \left(m_\pi^2 - x(1-x)p^2 \right) \Gamma \left(-1, \frac{m_\pi^2 - x(1-x)p^2}{\Lambda_\pi^2} \right) - \right. \\ &\quad \left. p^\mu p^\nu (1-2x)^2 \Gamma \left(0, \frac{m_\pi^2 - x(1-x)p^2}{\Lambda_\pi^2} \right) \right\} . \end{aligned} \quad (34)$$

Here Λ_π is a new cutoff which chops off the high momenta of the pion loop. This cutoff which is related to the size of the meson is independent of the quark loop cutoff Λ , which is the scale of spontaneous breaking of chiral symmetry. Again Λ_π indicates the range of validity of the effective meson theory. Since the effective meson theory arose from the bosonized NJL model in the gradient expansion we expect that $\Lambda_\pi < \Lambda$. This will come out later from the actual calculations. Let us also note that the self energy Σ_ρ is not transversal. This should come with no surprise since the ρ -meson does not couple to a conserved current. Nevertheless the longitudinal part will contribute neither to the electromagnetic pion form factor (since the electromagnetic current is conserved) nor to the $\pi^+\pi^-$ phase shifts (due to the kinematical structure of the $\rho\pi\pi$ vertex).

Figure 2: The electromagnetic form factor of the pion. The photon couples according to vector meson dominance to the ρ -meson and due to the $\rho\omega$ -mixing also to the ω -meson. The ρ -propagator is dressed by pion loops whereas the ω -propagator is given by a simple Breit-Wigner form. We neglected the error bars of the data points (represented by small circles) in order to show the quality of the fit. The experimental data are taken from [11].

Figure 3: The momentum dependence of the lowest order gradient expansion of the $\rho\omega$ -mixing $m_{\rho\omega}^2$ for three different constituent quark masses m_d . All other parameters are chosen to reproduce the empirical values for the low energy observables F_π, m_π, m_ρ and $g_{\rho\pi\pi}$. Then m_d remains as free parameter while $m_u = 300$ MeV is fixed. The bold lines shows $m_{\rho\omega}^2$ for $m_d = 301$ MeV, the thin line for $m_d = 302$ MeV and the dashed line is calculated with $m_d = 303$ MeV. For comparison, m_u has 300 MeV. The empirical value for $m_{\rho\omega}^2$ is only known on the vector meson mass shell $p^2 \approx (m_\rho^2 + m_\omega^2)/2 \approx 0.6 \text{ GeV}^2$ being $m_{\rho\omega}^2 \approx (-4.5 \pm 0.6) \times 10^3 \text{ MeV}^2$.

Figure 4: The $\pi^+\pi^-$ phase shifts in the vector-isovector channel calculated with the ρ -propagator (35) which is dressed by π -loops. Note that the ω -meson does not contribute here. The data (represented by circles) are taken from [12].

Ignoring for the moment the $\rho\omega$ -mixing but including the pion loop the transversal part of the ρ propagator would have been given by

$$D_\rho^{ab} = \frac{\delta^{ab}}{p^2 - m_\rho^2 - \Sigma_\rho} \quad . \quad (35)$$

Since we have ignored two pion loops and vector meson loops the ω -propagator does not receive a width. As the $\rho\omega$ -mixing is small the details of the ω -propagator will not be substantial

for the ρ -propagator. Since we are interested here in processes exclusively determined by the ρ -propagator we will not attempt a detailed evaluation of the ω -propagator but rather use the empirical Breit-Wigner form

$$D_\omega = \frac{1}{p^2 - m_\omega^2 - i\Gamma_\omega m_\omega} \quad , \quad (36)$$

where m_ω and Γ_ω are the empirical mass and width of the ω -meson. Note that the Breit-Wigner form ignores any momentum dependence of Γ_ω or m_ω , which would result in a two loop calculation from the Wess-Zumino term. We will see however later that a possible momentum dependence of the width Γ_ω or the mass m_ω is completely irrelevant for the ρ -propagator (since the $\rho\omega$ -mixing is so small that it matters only near the pole). Assuming the empirical Breit-Wigner shape (36) for the ω -propagator the $\rho\omega$ -mixing modifies the free ρ -propagator by the factor

$$\left(1 + \frac{1}{3} \frac{m_\omega^2}{m_\rho^2} \frac{m_{\rho\omega}^2}{p^2 - m_\omega^2 + im_\omega\Gamma_\omega} \right) \quad . \quad (37)$$

This is immediately seen by diagonalizing the $\rho\omega$ -mixing after having the free ω -lagrangian replaced by the Breit-Wigner form

$$\mathcal{L}_\omega = \frac{1}{2} \omega \left(\partial^2 + m_\omega^2 - im_\omega\Gamma_\omega \right) \omega \quad . \quad (38)$$

Therefore we finally obtain for the ρ -propagator, which includes both the pion loop as well as the $\rho\omega$ -mixing, the following expression

$$D_\rho^{ab} = \delta^{ab} \frac{1}{p^2 - m_\rho^2 - \Sigma_\rho} \left(1 + \frac{1}{3} \frac{m_\omega^2}{m_\rho^2} \frac{m_{\rho\omega}^2}{p^2 - m_\omega^2 + im_\omega\Gamma_\omega} \right) \quad . \quad (39)$$

Once the ρ propagator is known the electromagnetic form factor of the pion and the $\pi^+\pi^-$ phase shifts can be evaluated from (30) and (33).

4 Numerical results

In the actual numerical calculation let us first consider the effective meson theory derived from the bosonized NJL model in the low energy regime as a phenomenological meson model. That is we determine the involved parameters from the experimental meson data. Using the experimental values $m_\omega = 782\text{MeV}$ and $\Gamma_\omega = 843\text{MeV}$ we can adjust Λ_π, m_ρ and $g_{\rho\pi\pi}$ to fix the mass and width of the ρ -meson and the height of the electromagnetic form factor. Furthermore the $\rho\omega$ -mixing strength is adjusted to reproduce the fine structure of the electromagnetic pion form factor.

This yields

$$\begin{aligned}
\Lambda_\pi &= 663 \text{ MeV} \\
m_\rho^0 &= 922.5 \text{ MeV} \\
g_{\rho\pi\pi} &= 5.86 \\
m_{\rho\omega}^2 &= -4850 \text{ MeV}^2
\end{aligned}$$

The meson loop cutoff Λ_π is in fact smaller than the quark loop cutoff $\Lambda \sim 1.3 \text{ MeV}$ as anticipated. The bare ρ -mass m_ρ^0 is somewhat larger than the physical ρ -mass which is determined by the position of the peak in the electromagnetic form factor. Amazingly the $\rho\pi\pi$ coupling constant obtained from the fit of the electromagnetic form factor $g_{\rho\pi\pi} = 5.86$ agrees very well with the value predicted from the bosonized NJL model $g_{\rho\pi\pi} = 6$. The form factor obtained with these parameters is shown in figure (2). It amazingly well reproduces the experimental form factor. Note the kink in the form factor on the right hand side above the peak. Our analysis shows that this kink is exclusively generated by the $\rho\omega$ -mixing. This comes out also in the analysis of ref. [10].

Let us finally interpret the empirically determined value of the $\rho\omega$ -mixing in terms of the result from the bosonized NJL model. Using in (21) the empirical value for the mixing strength and fixing all parameters of the NJL model except the isospin breaking part $m_u - m_d$ in the standard fashion from $F_\pi, m_\pi, g_{\rho\pi\pi}$ yields for the splitting between the up and down current quark masses

$$m_d - m_u \approx (1 - 2) \text{ MeV} \quad , \quad (40)$$

which is not unrealistic. The momentum dependence of the mixing strength $m_{\rho\omega}^2$ for three different values of $\Delta\Sigma$ can be seen in figure (3). Notice that $m_{\rho\omega}^2$ vanishes for $p^2 = 0$ independent of the mass splitting of the quarks as a consequence of gauge symmetry.

Finally with the above determined ρ propagator we can now calculate the $\pi^+\pi^-$ phase shift in the vector-isovector channel in a parameter free way. The result is shown in figure (4).

The theoretically curve excellently reproduces the experimental data. Let us emphasize that our investigations show that the inclusion of the pion loop is crucial for obtaining the correct energy dependence of the phase shift. Note also that the $\rho\omega$ -mixing does not contribute to the phase shift but is crucial to obtain the fine structure of the electromagnetic form factor.

References

- [1] S. Weinberg, Phys. Rev. Lett. **18** (1967) 507; J. Gasser and H. Leutwyler, Ann. Phys. **158** (1984) 142.
- [2] D. Ebert and H. Reinhardt, Nucl. Phys. **B271** (1986) 188.

- [3] G. A. Miller, B. M. K. Nefkens and I. Slaus, Phys. Rep. **194** (1990) 1.
- [4] M. K. Volkov, Sov. J. Part. Nucl. **17** (1986) 186.
- [5] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **D122** (1961) 345; *ibid.* **D124** (1961) 246.
- [6] H. Reinhardt and B. V. Dang, Nucl. Phys. **A500** (1989) 563.
- [7] K. Fujikawa, Phys. Rev. **D21** (1980) 2848; **D23** (1981) 2262; **D29** (1984) 285.
- [8] J. Wess and B. Zumino, Phys. Lett. **37B** (1971) 95.
- [9] M. Bando, T. Kugo and K. Yamawaki, Phys. Rep. **164** (1988) 217.
- [10] J. Speth and B. C. Pearce, *The structure of Mesons and Nucleons*, proceedings of *4th International Spring Seminar on Nuclear Physics*, Amalfi, ed. A. Coviello, (1992) 29.
- [11] S. R. Amendolia et al., Phys. Lett. **B138** (1984) 454;
 S. R. Amendolia et al., Phys. Lett. **B146** (1984) 116;
 L. M. Barkov et al., Nucl. Phys. **B256** (1985) 365.
- [12] C. D. Froggatt and J. L. Petersen, Nucl. Phys. **B129** (1977) 89.

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9501333v1>

This figure "fig2-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9501333v1>

This figure "fig1-2.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9501333v1>

This figure "fig2-2.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9501333v1>